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Summary

The correct grasp of objects is a key aspect for the right fulfillment of a given task. In robotics, the development of grippers more and more complex and versatile, such as mechanical hands, augments the necessity for algorithms to automatically determine grasps, and in a parallel way, arises the need to quantify the quality of the generated grasps in order to optimize them. This work reviews the quality measures proposed in the grasp literature to quantify the grasp quality. We classify the quality measures into two groups according to the main aspect evaluated by the measure: the location of the contact points on the object or the hand configuration. We also review the approaches that combine different quality measures from the two previous groups to obtain a global quality measure.

KEYWORDS: Grasp, mechanical hands, grasp quality measures.

1 Introduction

Manipulation with complex grippers, such as mechanical hands, is an active research area in robotics. Different types of mechanical hands have been designed and built in the last two decades, and they have been classified based on the degree of anthropomorphism and dexterity level [1]. Grupen et al [2] review the technology required to build mechanical hands, and Bicchi [3] summarizes the evolution and state of the art in the field of robot hands. Apart from an efficient mechanical design, hands must have suitable tactile systems; Tegin and Wikander [4] make a survey on sensing systems, and Okamura et al. [5] highlight current accomplishments and challenges in hardware development for robotic manipulation.

Grasp and manipulation are the key functions of robot hands. The goal of a grasp is to achieve a desired object constraint in front of external disturbances or the object weight itself; in this line, robot grasp synthesis and fixture design for industrial parts are very related problems [6, 7]. Dexterous manipulation involves changing the object's position with respect to the hand without any external support. Table 1 summarizes previous surveys related to grasp and manipulation with robotic hands.

Two approaches have been used in grasp planning [8]: a physiological approach, trying to mimic the behavior of the human hand, and a mechanical approach, considering physical and mechanical properties involved in grasping. Cutkosky [14] has a remarkable work using the human hand as model for robot hands; he presents a taxonomy of the grasps used in manufacturing operations, classifying and choosing grasps according to the object attributes and task requirements. Grasp synthesis taking into account the grasp physical and mechanical properties requires the identification of such properties and the creation of suitable parameters to quantify them. The following are the main properties that have been identified hitherto in grasp bibliography [12]:

- *Disturbance resistance:* a grasp can stand disturbances in any direction if it fulfills one of the following conditions: form-closure (or complete kinematical restraint) when

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Table 1. Previous surveys related to grasp and manipulation with robotic hands.

Researchers	Year	Subject
Mishra and Silver [8]	1989	Grasp planning
Grupen et al. [2]	1989	Technology required to build mechanical hands
Murray et al. [9]	1994	Mathematical foundations of robotic manipulation
Bicchi [10]	1995	Form and force closure analysis
Mishra [11]	1995	Mathematical analysis of quality measures
Shimoga [12]	1996	Grasp synthesis algorithms and dexterity measures
Bicchi [3]	2000	State of the art in robot hands
Bicchi and Kumar [13]	2000	Robotic grasping
Okamura et al. [5]	2000	Actuation levels in dexterous manipulation
Lotti and Vassura [1]	2002	Classification and evaluation of robot hands
Tegin and Wikander [4]	2005	Tactile sensing in robotic manipulation

the positions of the fingers ensure object immobility, or force-closure when the forces applied by the fingers ensure the object immobility, i.e. the fingers can apply appropriate forces on the object to produce wrenches in any direction, so they can compensate any external wrench applied on the object (up to a certain magnitude). Bicchi [10] describes in detail these conditions, and Rimon and Burdick [15] address the relation of the contact surfaces curvature with form and force closure grasps.

- *Dexterity:* a grasp is dexterous if the kinematical relations between the hand and the object allow the hand to move the object in a compatible way with the task to be performed. In a general case, without task specifications, a grasp is considered dexterous if the manipulator device is able to move the object in any direction.

- *Equilibrium:* a grasp is in equilibrium when the resultant of forces and torques applied on the object (both by the fingers and by external disturbances) is null. An associated problem is the optimization of the finger forces on the object (making them as low as possible) in order to avoid damages on the object, provided that the object is properly restrained. The optimization is generally done minimizing an objective function, with constraints coming from the grasp problem formalism [16, 17, 18, 19].

- *Stability:* a grasp is stable if any object position or finger force error caused by a disturbance disappears in time after the disturbance vanishes. Thus, the grasp should generate restitution forces when it is moved away from the equilibrium [20, 21, 22].

Although these properties have been mainly established in the context of grasping, they are also valid in dexterous manipulation, where three levels of actuation were identified [5]: a high level based on discrete events (task planning including grasp choice), a medium level dealing with transitions between manipulation phases and event detections, and a low level regarding dynamics and continuous control of each phase in the manipulation.

Our work is focused on disturbance resistance and dexterity, and describes the quality measures related with these properties. These two properties are the most frequently considered in the grasp synthesis, i.e. in the determination of the finger contact points on the object and the determination of an appropriate hand configuration. Disturbance resistance and dexterity are considered at the highest level of dexterous manipulation [5]. In general, there is more than one grasp fulfilling these two properties, and an optimal one is chosen using a quality measure, i.e. a suitable parameter associated to those properties. We consider two groups of quality measures:

- Measures associated with the position of contact points.

- Measures associated with the hand configuration.

Some previous works include reviews of quality measures, either directly or as part of a more general survey on grasping aspects and problems, for instance, Mishra [11] presents the mathematical background required to study grasp quality measures considering disturbance

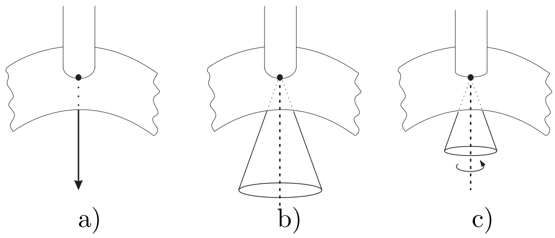

Fig. 1. Forces allowed in each type of contact: a) Point contact without friction; b) Point contact with friction; c) Soft contact.

resistance, Shimoga [12] reviews grasp synthesis algorithms, including quality measures associated with the hand configuration, and Bicchi and Kumar [13] describe the state of the art in robotic grasping, including a discussion about the relevance of grasp quality measures. Our work compiles the quality measures presented hitherto, and classifies them into the two groups previously stated.

After this introduction the report is structured as follows. Section 2 resumes the basic background necessary to formalize the grasp quality measures. Section 3 presents the quality measures associated with the position of the contact points, and Section 4 presents those associated with the hand configuration. Section 5 reviews the approaches that combine different quality measures from the two previous groups to obtain a global quality measure. Finally, Section 6 presents the conclusions of this work.

2 Basic background

This Section presents the basic background used in the rest of the paper, describing the main variables involved in grasping and manipulation and the relations between them.

2.1 Force and velocity modelling

Consider a coordinate system located at the object center of mass (CM) to describe the positions $\boldsymbol{p}$ of the contact points and the forces applied on the grasped object. A force $\boldsymbol{f}_i$ applied on the object at the point $\boldsymbol{p}_i$ generates a torque $\boldsymbol{\tau}_i = \boldsymbol{p}_i \times \boldsymbol{f}_i$ with respect to CM. The force and the torque are grouped together in a wrench vector $\boldsymbol{\omega}_i = (\boldsymbol{f}_i, \lambda\boldsymbol{\tau}_i)^T$ (also known as generalized force vector), with λ being a constant that defines the metric of the wrench space. In order to make the space isotropous in terms of energy, λ is selected equal to the radius of gyration of the object, nevertheless, in some cases the quality measure is independent of λ and it is not explicitly considered or considered as $\lambda = 1$. $\boldsymbol{\omega} \in \Re^d$ and the dimension of the wrench space is $d = 3$ for the bidimensional physical space (planar problem with 2D objects) and $d = 6$ for 3D objects in the tridimensional physical space.

The movement of the object is described through the translational velocity of CM, $\boldsymbol{V}_{CM}$, and the rotational velocity of the object with respect to CM, $\boldsymbol{\psi}$; the two velocities are represented as a twist (also known as generalized velocity) $\dot{\boldsymbol{x}} = (\boldsymbol{V}_{CM}, \boldsymbol{\psi})^T$, with $\dot{\boldsymbol{x}} \in \Re^d$.

A fingertip force $\boldsymbol{f}_i$ is produced by the torques $\boldsymbol{T}_{ij}$, $j = 1, ..., m$, applied at each of the m joints of finger i. In a hand with n fingers, a vector $\boldsymbol{T} = \left[\boldsymbol{T}_{1j}^T \dots \boldsymbol{T}_{nj}^T\right]^T \in \Re^{nm}$ is defined to group all the torques applied at the hand joints. Also, the velocities in the finger joints, $\dot{\boldsymbol{\theta}}_{ij}$, are grouped in a single vector $\dot{\boldsymbol{\theta}} = \left[\dot{\boldsymbol{\theta}}_{1j}^T \dots \dot{\boldsymbol{\theta}}_{nj}^T\right]^T \in \Re^{nm}$.

2.2 Contact modelling

The forces applied by the fingertips at the contact points can act only against the object (positivity constraint), and the type of contact between the fingertips and the object can be (Figure 1):

- *Point contact without friction:* the contact occurs in a frictionless point, therefore the exerted force on the object is normal to the contact surface.
- *Point contact with friction (rigid contact):* the contact occurs in a frictional point, therefore the exerted force has a component normal to the contact surface and may have another one tangential to the contact surface. Several models have been proposed to represent friction [23]. The most common one is the Coulomb´s friction model stating that slippage is avoided when $\boldsymbol{f}^t \leq \mu \boldsymbol{f}^n$, where $\boldsymbol{f}^n$ is the normal force, $\boldsymbol{f}^t$ is the tangential force, and μ is the friction coefficient. In the tridimensional physical space this friction model is non-linear, and sometimes it is not easy to manage it. Two approaches have been proposed to deal with this problem:

  a) Linearization of the friction cone with a conservative approach that approximates it with a convex cone of s faces (the largest s the better the approximation, but also the largest the computational cost of any operation involving it).

  b) Definition of a particular matrix M, whose elements are functions of the contact force components, such that the positiveness of its eigenvalues implies that the contact force satisfies the positivity and the friction constraints [24, 25] (the use of these matrices allows a generic formalism to check whether a given force lies or not inside the friction cone).
- *Soft contact:* the contact occurs in a region, and there is friction between the parts. This type of contact allows the application of the same forces as the rigid contact plus a torque around the direction normal to the contact zone, therefore the model is valid only for 3D objects. In this case, Coulomb´s friction model assumes that friction limits due to torsion and shear forces are independent; nevertheless, it has been experimentally shown that those effects are coupled [23], and the coupling has been modelled with a linear or an elliptical relation between torsional and shear frictional forces [17, 26].

The number r of non-null independent components of the possible applied wrenches at each contact point depend on the type of contact: $r = 1$ for the point contact without friction (i.e. all the possible applied forces have the same direction); for the rigid contact $r = 2$ in the bidimensional physical space and $r = 3$ in the tridimensional physical space (i.e. the dimension of the friction cone), and $r = 4$ for the soft contact (i.e. the dimension of the friction cone plus the torque around the friction cone axis).

Forces and velocities for each fingertip can also be expressed in a reference system defined at each contact point. The vector $\boldsymbol{f} = \left[\boldsymbol{f}_{1k}^T \dots \boldsymbol{f}_{nk}^T\right]^T \in \Re^{nr}$ $(k = 1, ..., r)$ groups all the applied force components at all the contact points; in a similar way, the vector $\boldsymbol{v} = \left[\boldsymbol{v}_{1k}^T \dots \boldsymbol{v}_{nk}^T\right]^T \in \Re^{nr}$ contains all of the velocity components of the fingertips.

2.3 Relations between forces and velocities

Forces and velocities associated with the object, the hand and the contact points satisfy certain relations (for instance, the net wrench on the object is related to the forces produced on the fingertips transferred through the contact points, which in turn depend on the torques applied on the finger joints). These relations, illustrated in Figure 2, are the followings.

Fingertips forces $\boldsymbol{f}$ and velocities $\boldsymbol{v}$ are related to torques and velocities of the finger joints through the hand Jacobian, $J_h = \text{diag}\left[J_1, \dots, J_i\right] \in \Re^{nr \times nm}$, where $J_i \in \Re^{r \times m}$, $i = 1, \dots, n$, is the Jacobian of finger i that relates its joint variables with its fingertip variables. Relations are given by:

$$\boldsymbol{T} = J_h^T \boldsymbol{f} \tag{1}$$

$$\boldsymbol{v} = J_h \dot{\boldsymbol{\theta}} \tag{2}$$

The relation between fingertip forces and the net wrench applied on the object $\boldsymbol{\omega}$, and the relation between velocities at the contact points and the twist $\dot{\boldsymbol{x}}$ are established through the grasp matrix $G \in \Re^{d \times nr}$ (sometimes referred as grasp Jacobian or grasp map):

$$\boldsymbol{\omega} = G \boldsymbol{f} \tag{3}$$

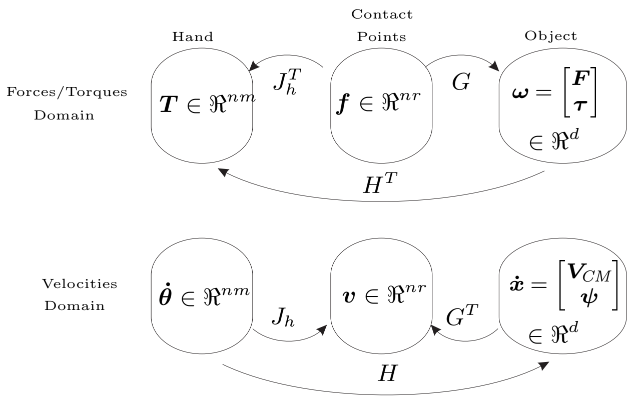

Fig. 2. Relationships between the grasp force and velocity domains.

$$\boldsymbol{v} = G^T \dot{\boldsymbol{x}} \tag{4}$$

Finally, the relationship between the hand space and the object space is given by the hand-object Jacobian $H \in \Re^{d \times nm}$, which is obtained as

$$H = (G^+)^T J_h \tag{5}$$

with G^+ being the generalized inverse of G (note that usually G is not a square matrix). Thus,

$$\boldsymbol{T} = H^T \boldsymbol{\omega} \tag{6}$$

$$\dot{\boldsymbol{x}} = H \dot{\boldsymbol{\theta}} \tag{7}$$

More about these relations is given by Murray et al. [9].

2.4 Duality between force and velocity

A mechanical hand can be considered as a mechanical transformer with joint forces or velocities as inputs and generalized force or velocity as outputs [27]; the transformation is given by (6) and (7).

Consider a sphere of unitary radius in the velocity domain of the hand joints, given by

$$\left\| \dot{\theta} \right\|^2 = \dot{\theta}_1^2 + \dot{\theta}_2^2 + \cdots + \dot{\theta}_{nm}^2 \leq 1 \tag{8}$$

Equation (7) maps this sphere into an ellipsoid in the generalized velocity domain of the object, known as the velocity ellipsoid:

$$\dot{\boldsymbol{x}}^T \left(H H^T \right)^{-1} \dot{\boldsymbol{x}} \leq 1 \tag{9}$$

This ellipsoid represents the gain in each direction of the generalized velocity domain of the object when a unitary velocity is applied in the velocity domain of the hand joints.

Now consider an unitary sphere in the torque domain of the hand joints

$$\|T\|^2 = T_1^2 + T_2^2 + \cdots + T_{nm}^2 \leq 1 \tag{10}$$

Equation (6) maps such sphere into an ellipsoid in the generalized force domain, known as the force ellipsoid:

$$\boldsymbol{\omega}^T \left(H H^T \right) \boldsymbol{\omega} \leq 1 \tag{11}$$

This ellipsoid represents the gain in each direction of the generalized force domain of the object when a unitary velocity is applied in the torque domain of the hand joints.

The matrices $\left(H H^T \right)^{-1}$ and $H H^T$, defining respectively the velocity and force ellipsoids, are the inverse of each other, so they have the same eigenvalues and eigenvectors and also the same volume (the two ellipsoids receive the generic denomination of manipulability ellipsoid [28]). This implies that the principal axes for the two ellipsoids are coincident, but their lengths are in inverse proportion, i.e. the direction with the maximum transmission ratio for velocities has the minimum transmission ratio for force, and vice versa. This leads to the following reasonings [27]:

- The optimal direction (largest gain) to apply a force on the object is the direction of the major axis of the force ellipsoid, because the transmission ratio is maximum. The same reasoning is valid for velocities using the major axis of the velocity ellipsoid.
- The most accurate control of force or velocity is along the direction of the minor axis of the force or velocity ellipsoids respectively, because the transmission ratio is a minimum.

3 Quality measures associated with the position of the contact points

This first group of quality measures includes those that only take into account the object properties (shape, size, weight), friction constraints and form and force closure conditions to quantify the grasp quality. These measures are classified into three subgroups: one considering only algebraic properties of the grasp matrix G, another one considering geometric relations in the grasp (thus assuming in both cases that all of the fingers can apply forces without a magnitude limit), and a third subgroup of measures that considers limits in magnitudes of the finger forces.

3.1 Measures based on algebraic properties of the grasp matrix G

This subgroup of measures includes those that only take into account the grasp matrix G to quantify the grasp quality. The definition of these measures does not consider any constraint on the forces at the contact points.

3.1.1 Minimum singular value of G

A full-rank grasp matrix $G \in \Re^{d \times nr}$ has d singular values given by the positive square roots of the eigenvalues of GG^T. When a grasp is in a singular configuration (i.e. when at least one degree of freedom is lost due to hand configuration), at least one of the singular values goes to zero. The smallest singular value of the grasp matrix G, $\sigma_{min}(G)$, is a quality measure that indicates how far is the grasp configuration from falling into a singular configuration [29], this is

$$Q = \sigma_{\min}(G) \tag{12}$$

The largest $\sigma_{\min}(G)$, the better the grasp. At the same time the largest the $\sigma_{\min}(G)$ the largest the minimum transmission gain from the forces $\boldsymbol{f}$ at the contact points to the net wrench $\boldsymbol{\omega}$ on the object, which is also used as grasp optimization criterion [30].

A drawback of this quality measure is that it is not invariant under a change in the reference system used to compute torques.

3.1.2 Volume of the ellipsoid in the wrench space

The effect of the grasp matrix G in the transformation relations given by equations (3) and (4) is visualized through ellipsoids, similar to those presented in Subsection 2.4. The equation (3) maps a sphere of unitary radius in the force domain of the contact points into an ellipsoid in the wrench space. The global contribution of all the contact forces can be considered using as quality measure the volume of this ellipsoid, this is [31]

$$Q = k\sqrt{\det(GG^T)} = k\,(\sigma_1 \sigma_2 \cdots \sigma_d) \tag{13}$$

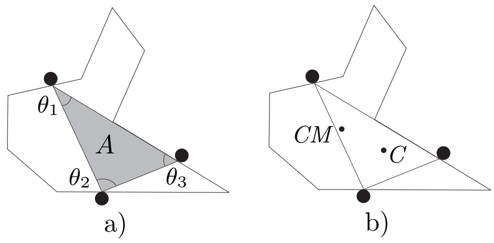

Fig. 3. Elements used in the measures based on geometrical relations: a) Internal angles and area of the grasp polygon; b) Distance between the centroid C of the grasp polygon and the center of mass CM of the object.

with k being a constant and σ_1, σ_2,…, σ_d denoting the singular values of the grasp matrix G. This quality measure considers all the singular values with the same weight and must be maximized to obtain the best grasp.

This measure is invariant under a change in the torque reference system.

3.1.3 Grasp isotropy index

This criterion looks for an uniform contribution of the contact forces to the total wrench exerted on the object, i.e. tries to obtain an isotropic grasp where the magnitudes of the internal forces are similar. The quality measure, called grasp isotropy index [30], is defined as:

$$Q = \frac{\sigma_{\min}(G)}{\sigma_{\max}(G)} \tag{14}$$

with $\sigma_{\max}(G)$ and $\sigma_{\min}(G)$ being the maximum and minimum singular value of G. This index approaches to 1 when the grasp is isotropic (optimal case), and falls to zero when the grasp is close to a singular configuration.

3.2 Measures based on geometric relations

This subgroup of measures considers those that quantify the grasp quality through the evaluation of certain geometric relations of the contact points.

3.2.1 Shape of the grasp polygon

In planar grasps, either of 2D or 3D objects, it is desirable that the contact points are distributed in an uniform way on the object surface, this improves the grasp stability [30, 32]. An index to quantify the uniform distribution of the fingers on the object compares how far are the internal angles of the grasp polygon (whose vertices are the contact points on the object –see an example in Figure 3a–) from those of the corresponding regular polygon . The quality of the grasp under this criterion, called the stability grasp index [30], is given by

$$Q = \frac{1}{\theta_{\max}} \sum_{i=1}^{n} \left|\theta_i - \bar{\theta}\right| \tag{15}$$

where n is the number of fingers, θ_i the internal angle at vertex i of the contact polygon, $\bar{\theta}$ is the average internal angle of the corresponding regular polygon (given in degrees by $\bar{\theta} = 180(n-2)/n$), and $\theta_{\max} = (n-2)(180-\bar{\theta}) + 2\bar{\theta}$ is the sum of the internal angles when the polygon has the most ill conditioned shape (i.e. degenerates into a line and the internal angles are either 0 or π). The stability index is minimum when the contact polygon is regular; for instance, in a three finger grasp the grasp is optimum when the contact polygon is an equilateral triangle [32, 33].

3.2.2 Area of the grasp polygon

In three-finger grasps, a larger triangle formed by the contact points on the object $\boldsymbol{p}_1$, $\boldsymbol{p}_2$ and $\boldsymbol{p}_3$ (Figure 3a) gives a more robust grasp, i.e. with the same finger force, the grasp can resist larger external torques [33, 34]. Thus the area of the grasp triangle is also used as quality measure (either of 2D or 3D objects), i.e.

$$Q = \text{Area}(\text{Triangle}(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3)) \tag{16}$$

Although there are no related works, this idea could be extended to grasps of 2D and 3D objects involving more than three fingers maximizing the area or the volume of the convex hull of the contact points.

3.2.3 Distance between the centroid of the contact polygon and the center of mass of the object

The effect of inertial and gravitational forces on the grasp is minimized when the distance between the center of mass of the object, CM, and the centroid C of the contact polygon (for 2D objects) or polyhedron (for 3D objects) is minimized (Figure 3b). Then, this distance is also used as a grasp quality measure, both for 2D objects [35] as well as for 3D objects [36, 37, 38],i.e.

$$Q = \| CM - C \| \tag{17}$$

3.2.4 Margin of uncertainty in the finger positions

For 2D objects, the space defined by the n parameters representing the possible contact points of n fingers on the object boundary is called the contact space (or grasp space), and the subset of the contact space where force-closure grasps are obtained is called the force-closure space, FCS [39]. There are different proposals for the computation of the force-closure space considering polygonal objects and any number of fingers, with or without friction [39, 40, 41], where it is stated that the FCS is the union of a set of convex polyhedra CP_i.

Considering the existence of uncertainty in the actual positioning of the fingers, the more far away from the boundary of the FCS the more secure will be the grasp. With this criterion, given a grasp represented by a point P in the contact space, it was proposed as grasp quality measure the radius of the largest hypersphere centered at P and fully contained in one of the convex polyhedra CP_i that form the FCS. The quality index is given by

$$Q = \min_{P_j \in \partial \text{CP}_i} \| P - P_j \| \tag{18}$$

with ∂CP_i being the boundary of the convex polyhedron i in the FCS. An example for three fingers, and therefore tridimensional contact space and force-closure space, is shown in Figure 4.

This measure has not been applied to 3D objects due to the complexity of the resulting grasp space (note that two parameters are needed to fix the position of each finger on the object surface).

3.2.5 Independent contact regions

Another approach dealing with uncertainty in the positioning of the fingers on the object is based on the computation of regions on the object boundary such that if each finger is positioned inside one of these regions a force closure grasp is obtained, with independence of the exact contact points within the regions. These regions are called independent contact regions [42], and they define a parallelepiped aligned with the reference axis in the grasp space and fully contained in the force-closure space. The larger the size of the regions (i.e the edges of the parallelepiped) the larger the set of acceptable correct positions that produce a force closure grasp (helping, for instance, in finding reachable contact points for a given mechanical hand). With this criterion, when independent contact regions are used the

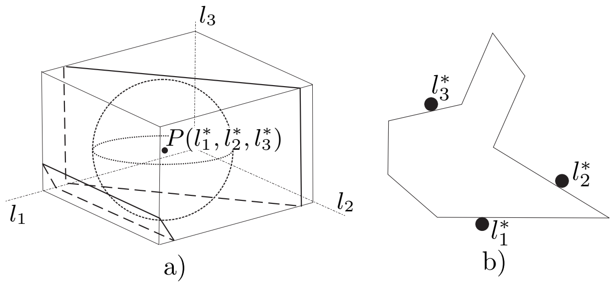

Fig. 4. Maximization of the margin of uncertainty: a) Maximum hypersphere in the FCS; b) Optimum grasp P.

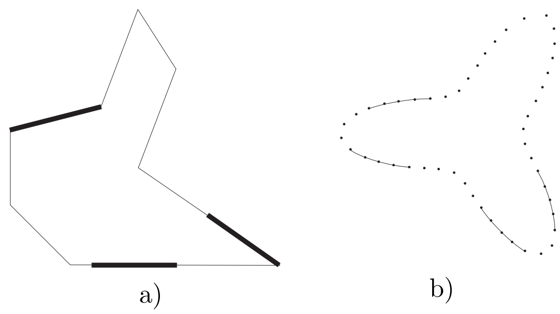

Fig. 5. Examples of independent contact regions: a) Polygonal objects and three fingers; b) Non-polygonal discrete objects and four fingers.

quality of a grasp can be associated with the size $L_{\min}$ of the smallest independent region (i.e the length of the shortest edge of the parallelepiped) [36], i.e.

$$Q = L_{\min} \tag{19}$$

This criterion was also developed for grasps of non-polygonal 2D objects with two fingers [43], in this case the grasp space is bidimensional, the force-closure space is limited by curves, and the independent regions are obtained by maximizing the shortest edge of an inscribed rectangle. The criterion was also adapted to the case of 2D non-polygonal objects represented by a discrete boundary, i.e. by a finite number of sampled points [44]. In this case, the quality is associated with the number of sampled points enclosed in the independent contact regions. Figure 5 shows two examples of independent contact regions.

The grasp will allow the maximum error in the finger locations if each finger is nominally positioned in the center of each independent contact region. Then, another quality measure, proposed for polyhedral objects and called uncertainty grasp index or grasp margin [30, 34], is given by the sum of the distances between each of the i-th actual contact points (x_i, y_i, z_i) and the center of the corresponding independent contact region (x_{i0}, y_{i0}, z_{i0}), i.e.

$$Q = \frac{1}{n} \sum_{i=1}^{n} \left(\sqrt{(x_i - x_{i0})^2 + (y_i - y_{i0})^2 + (z_i - z_{i0})^2} \right) \tag{20}$$

This index reaches the optimal value (zero) when all the fingers are located in the center of each independent contact region.

3.3 Measures considering limitations on the finger forces

The previous subgroups of quality measures include those related with the geometric location of the contact points, but they do not consider any limit on the magnitude of the forces applied by the fingers. Thus, even when the obtained force-closure grasps can resist external perturbation wrenches with any direction, nothing is said about the magnitude of the perturbation that can be resisted. This means that in some cases the fingers may have to apply extremely large forces to resist small perturbations. Then, the quality of a grasp can also be measured considering the module of the perturbation wrench that the grasp can resist when there are limits in the forces that the fingers can apply. This Subsection includes the measures that consider this aspect.

3.3.1 Largest-minimum resisted wrench

The most common constraints on the finger forces $\boldsymbol{f}_i$ are:

- The module of the force applied by each finger is limited, which corresponds to a limited independent power source (or transmission) for each finger. In order to simplify the formalism, it is assumed that all the fingers forces have the same limit and that it is normalized to 1, i.e. $\|\boldsymbol{f}_i\| \leq 1$, $i = 1, ..., n$.

  Considering that the friction cone at contact point $\boldsymbol{p}_i$ is approximated by a pyramid with s edges (Subsection 2.2), the force $\boldsymbol{f}_i$ applied by the finger can be expressed as a positive linear combination of unitary forces $\boldsymbol{f}_{ij}$, $j = 1, ..., s$, along the pyramid edges (usually called primitive forces), and the wrench $\boldsymbol{\omega}_i$ produced by $\boldsymbol{f}_i$ at $\boldsymbol{p}_i$ (Subsection 2.1) can be expressed as a positive linear combination of the wrenches $\boldsymbol{\omega}_{i,j}$ produced by $\boldsymbol{f}_{ij}$ (usually called primitive wrenches). Now, n fingers produce a resultant wrench on the object given by

  $$\begin{gathered} \boldsymbol{\omega} = \sum_{i=1}^{n} \boldsymbol{\omega}_i = \sum_{i=1}^{n} \sum_{j=1}^{s} \alpha_{i,j} \boldsymbol{\omega}_{i,j} \\ \text{with } \alpha_{i,j} \geq 0, \ \sum_{j=1}^{s} \alpha_{i,j} \leq 1 \end{gathered} \tag{21}$$

  Considering the possible variations of $\alpha_{i,j}$, the set $\mathcal{P}$ of possible resultant wrenches on the object is the convex hull of the Minkowski sum of the primitive wrenches $\boldsymbol{\omega}_{i,j}$:

  $$\mathcal{P} = \text{ConvexHull}\left(\bigoplus_{i=1}^{n} \{\boldsymbol{\omega}_{i,1}, \ldots, \boldsymbol{\omega}_{i,s}\} \right) \tag{22}$$

- The sum of the modules of the forces applied by the n fingers is limited, which corresponds to a limited common power source for all the fingers. Assuming again a limit normalized to 1, the constraint is $\sum_{i=1}^{n} \|\boldsymbol{f}_i\| \leq 1$.

  Modelling the friction cone as before, the resultant wrench on the object is given by

  $$\begin{gathered} \boldsymbol{\omega} = \sum_{i=1}^{n} \sum_{j=1}^{s} \alpha_{i,j} \boldsymbol{\omega}_{i,j} \\ \text{with } \alpha_{i,j} \geq 0, \ \sum_{i=1}^{n} \sum_{j=1}^{s} \alpha_{i,j} \leq 1 \end{gathered} \tag{23}$$

  and now the set $\mathcal{P}$ of all the possible resultant wrenches on the object is the convex hull of the primitive wrenches:

  $$\mathcal{P} = \text{ConvexHull}\left(\bigcup_{i=1}^{n} \{\boldsymbol{\omega}_{i,1}, \ldots, \boldsymbol{\omega}_{i,s}\} \right) \tag{24}$$

The set $\mathcal{P}$ of possible resultant wrenches produced by the fingers on the object is sometimes called as *Grasp Wrench Space* [45, 46].

There are other proposed constraints on the finger forces [11], like for instance $\sum_{i=1}^{n} \|\boldsymbol{f}_i\|^2 \leq 1$; however, the physical interpretations are not as evident as in the previous ones and they have not been widely implemented.

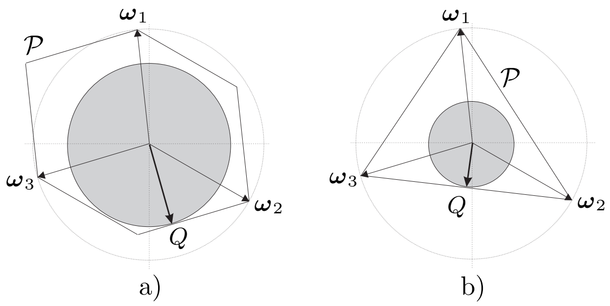

Fig. 6. Qualitative bidimensional example of the grasp quality using three fingers and: a) a limit in the module of each force; b) a limit in the sum of the modules of the applied forces.

With the force constraints, it was proposed as quality measure the largest perturbation wrench that the grasp can resist with independence of its direction; geometrically, the quality is equivalent to the radius of the largest ball centered at the origin of the wrench space and fully contained in $\mathcal{P}$, or, in other words, it is also equivalent to the distance from the origin of the wrench space to the closest face of $\mathcal{P}$ [47, 48]. The quality measure can be expressed as

$$Q = \min_{\boldsymbol{\omega} \in \partial \mathcal{P}} \|\boldsymbol{\omega}\| \tag{25}$$

with $\partial \mathcal{P}$ being the boundary of $\mathcal{P}$. This quality measure is sometimes referred as the criterion of the largest ball. This is one of the most popular quality measures, and is used in several works [49, 50, 51].

An optimal grasp under a force constraint is not necessarily optimal under another one; Figure 6 qualitatively illustrates with a bidimensional example the constraints on the finger forces described in equations (21) and (23), the sets of possible wrenches, and the resulting qualities in each case.

3.3.2 Variations around the criterion of the largest ball

The quality measure given by equation (25) is interpreted using metric L_2; some works [11] propose the use of other metrics, such as L_1 or L_∞, but implementations or detailed discussions about their use were not reported in the bibliography.

The radius of the largest ball inscribed in $\mathcal{P}$ depends on the choice of the origin of the reference system used to compute torques, i.e. the *CM* of the object (which sometimes can not be precisely located), and an optimum grasp with respect to a reference system could not be optimal with respect to another one. Different alternatives have been proposed to avoid this effect, like the use as quality measure of the radius of the largest ball with respect to all the possible choices of reference systems [52], however, this measure has not been widely considered due to its high computational cost. Another alternative quality measure is the volume of $\mathcal{P}$ [53], which is constant independently of the reference system used to compute torques.

A different approach replaces the ball by a convex compact set C that includes the origin [54, 55] and may represent expected perturbations, then the quality measure (called the Q-distance by the authors) is the largest scale factor ρ that makes C be fully contained in $\mathcal{P}$. In order to facilitate the implementation of this measure, C is taken as a convex polyhedron in the wrench space. The quality measure is given by

$$Q = \max_{\rho C \subset \mathcal{P},\, \rho \geq 0} \rho \tag{26}$$

With the aim of avoiding the definition of a metric to join forces and torques in the wrench space (the factor λ introduced in Subsection 2.1), the following optimality criterion for grasp synthesis was proposed [33]: first, the grasps that better resists pure forces are computed and then, from them, the grasps with the best resistance to pure torques are chosen. The quality measures used in each step are

$$Q^f = \min_{\boldsymbol{f} \in \partial \mathcal{P}^f} \|\boldsymbol{f}\| \tag{27}$$

$$Q^\tau = \min_{\boldsymbol{\tau} \in \partial \mathcal{P}^\tau} \|\boldsymbol{\tau}\| \tag{28}$$

with $\partial \mathcal{P}^f$ and $\partial \mathcal{P}^\tau$ the boundaries of the sets of possible resultant forces and resultant torques, respectively, that the fingers can generate on the object.

3.3.3 Normal directions at the contact points

The sum of the components of the applied forces normal to the object boundary is indicative of the internal forces that the object withstands when an external disturbance is applied. Then, a quality measure is defined as the sum of the modules of the normal components of the applied forces required to achieve an expected demanding wrench $\boldsymbol{\omega}_0$ (the own object weight is the main considered disturbance); this measure is called Max-Normal-Grasping-Force [56] or grasp effort[57] and must be minimized to get an optimum grasp. As a difference with the criterion of the largest ball, this quality measure fixes beforehand the external wrench to be resisted and considers then the required forces.

$$Q = \min_{G\boldsymbol{f} = \boldsymbol{\omega}_0, M > 0} \sum_{i=1}^{n} \boldsymbol{f}_i^n \tag{29}$$

with $\boldsymbol{f}_i^n$ being the normal component of the force $\boldsymbol{f}_i$, and $M > 0$ representing that M is positive definite and therefore implying the satisfaction of the positivity and friction constraints (Subsection 2.2).

Another approach takes into account that if the finger forces applied at each contact point in absence of perturbations are close to the directions normal to the object boundary, the applied forces can vary in a larger range of directions to deal with external perturbations, while if the finger forces are close to the limit of the friction cone the fingers could easily slip when trying to keep the force-closure grasp. This effect is considered in another grasp quality measure called *Min-Analytic Center* [56] given by

$$Q = \min_{G\boldsymbol{f} = \boldsymbol{\omega}_0, M > 0} \log \det M^{-1} \tag{30}$$

The measure tends to infinity when any contact force approaches the boundary of its friction cone [58], thus, the smaller Q the better the grasp.

3.3.4 Task oriented measures

When there is a detailed description of the task to be performed, the grasp quality measure can quantify the ability of the grasp to counteract the expected disturbances during the task execution. The task can be characterized by a set of wrenches that must be applied on the object to achieve a given objective and a set of expected disturbance wrenches that the object must withstand while being manipulated. All these wrenches form a task polytope (called *Task Wrench Space* by some authors [59, 45]), that is commonly approximated with an ellipsoid $\mathcal{E}$ [31]. The quality measure is the scale factor ρ required to obtain the largest ellipsoid centered at the origin and fully contained in $\mathcal{P}$ (so the largest the ρ the better the grasp),

$$Q = \max_{\rho \mathcal{E} \subset \mathcal{P},\, \rho \geq 0} \rho \tag{31}$$

Figure 7 shows a comparison of this measure with the radius of the largest ball inscribed in $\mathcal{P}$. While the ball assumes that the probability for every disturbance direction is equal, the ellipsoid takes into account the most demanding wrench directions to complete the task. Several implementations of this quality measure have been presented [59, 60].

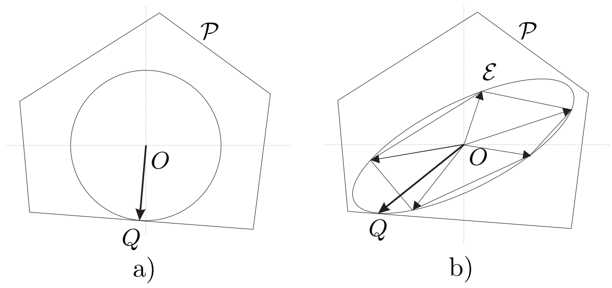

Fig. 7. Examples for the same applied forces: a) Largest ball contained in $\mathcal{P}$; b) Task oriented quality measure.

4 Quality measures associated with the hand configuration

This second group of quality measures includes those that consider the hand configuration to estimate the grasp quality. The basic ideas behind some of these measures have been presented in Subsection 3.1, where the quality measures depended on the properties of matrix G, while here the measures require the hand-object Jacobian H to quantify the quality.

4.1 Distance to singular configurations

In order to keep redundant arms away from singular configurations, it is desirable to maximize the smallest singular value σ_{min} of the manipulator Jacobian [61]. The same idea is applied to grasps with mechanical hands using the hand-object Jacobian H [12], which in a singular grasp configuration has at least one of the singular values equal zero. Therefore, by using $\sigma_{min}(H)$ as a quality measure, maximizing the quality is equivalent to choose a grasp configuration far away from a singular one. Then,

$$Q = \sigma_{min}(H) \tag{32}$$

4.2 Volume of the manipulability ellipsoid

The measure $\sigma_{min}(H)$ considers only one singular value of H, which may be similar for two different grasp configurations. In order to consider all the singular values of H, the volume of the manipulability ellipsoid (Subsection 2.4) is proposed as quality measure [62]. Let $\sigma_1, \sigma_2, ..., \sigma_r$ be the singular values of H. The grasp quality (i.e. the volume of the manipulability ellipsoid) is

$$Q = k\sqrt{\det(HH^T)} = k\,(\sigma_1\sigma_2...\sigma_r) \tag{33}$$

where k is a constant. The quality is therefore proportional to the product of all the singular values, and maximizing the determinant of HH^T maximizes the volume of the ellipsoid. Physically, this means that the same velocities in the finger joints in two different grasp configurations produce a largest velocity of the grasped object in the configuration with the largest quality [12].

4.3 Uniformity of transformation

The transformation between the velocity domain in the finger joints and the velocity domain of the object is uniform when the contribution of each joint velocity is the same in all the components of the object velocity; in this case the hand can move the object in any direction with the same gain, which is a good manipulation ability. The condition number n_c of the finger Jacobian is a measure of such ability [63],

$$n_c(J_i) = \left\|J_i\right\| \left\|J_i^{-1}\right\| \tag{34}$$

with $\|\cdot\|$ representing a matrix norm. This concept was used to measure the quality of a grasp considering the condition number of H as a measure of the manipulability of the grasped object [12]. The condition number of H can be computed from its maximum and minimum singular values ($\sigma_{\max}$ and $\sigma_{\min}$) as,

$$Q = n_c(H) = \frac{\sigma_{\max}(H)}{\sigma_{\min}(H)} \tag{35}$$

When $n_c(H) = 1$ the columns of H are vectors orthogonal to each other and with the same module, indicating an uniform transformation. The quality of a grasp will be better when the grasp configuration gives n_c as close as possible to 1.

4.4 Positions of the finger joints

An useful criterion to choose configurations in redundant robot arms is to look for configurations whose joints are as far as possible from their physical limits, i.e. with the joint positions as close as possible to the center of their ranges [64], and the same idea is applied to mechanical hands [12]. The index used to quantify the joint angle deviations is

$$Q = \sum_{i=1}^{nm} (\theta_i - \theta_{0i})^2 \tag{36}$$

where θ_i and θ_{0i} are, respectively, the actual and the middle-range position of the i-th joint (obviously, the index is simplified when $\theta_{0i} = 0$). The minimization of Q implies a grasp configuration with the joint positions close to the middle-range reference position.

The measure (36) could be redefined to appropriately weight the different range of each joint as

$$Q = \sum_{i=1}^{nm} \left(\frac{\theta_i - \theta_{0i}}{\theta_{\max_i} - \theta_{\min_i}} \right)^2 \tag{37}$$

4.5 Task compatibility index

This index considers the requirements of the task in the measure of the grasp quality [27, 65]; if there are directions of wrenches more likely to be applied on the object, the grasp should try to assure the maximum transformation ratio along these directions. Consider an unitary vector $\hat{\boldsymbol{\omega}}_i$ in the wrench space with the direction of a force requirement, and the distance α_i from the origin to the surface of the force ellipsoid in the direction $\hat{\boldsymbol{\omega}}_i$. Thus, $\alpha_i \hat{\boldsymbol{\omega}}_i$ represents a point on the force ellipsoid, satisfying the following relation:

$$(\alpha_i \hat{\boldsymbol{\omega}}_i)^T \left(HH^T\right) (\alpha_i \hat{\boldsymbol{\omega}}_i) = 1 \tag{38}$$

with

$$\alpha_i = \left[\hat{\boldsymbol{\omega}}_i^T \left(HH^T\right) \hat{\boldsymbol{\omega}}_i\right]^{-1/2} \tag{39}$$

In a similar way, consider an unitary vector $\hat{\dot{\boldsymbol{x}}}_j$ with the direction of a velocity requirement, and the distance β_j from the origin to the surface of the velocity ellipsoid in the direction $\hat{\dot{\boldsymbol{x}}}_j$, so $\beta_j \hat{\dot{\boldsymbol{x}}}_j$ satisfies the following relation:

$$\left(\beta_j \hat{\dot{\boldsymbol{x}}}_j\right)^T \left(HH^T\right)^{-1} \left(\beta_j \hat{\dot{\boldsymbol{x}}}_j\right) = 1 \tag{40}$$

with

$$\beta_j = \left[\hat{\dot{\boldsymbol{x}}}_j^T \left(HH^T\right)^{-1} \hat{\dot{\boldsymbol{x}}}_j\right]^{-1/2} \tag{41}$$

With these elements, the task compatibility index is defined as

$$Q = \sum_{i=1}^{s} \gamma_i \alpha_i^{\pm 2} + \sum_{j=s+1}^{z} \gamma_j \beta_j^{\pm 2}$$
$$= \sum_{i=1}^{s} \gamma_i \left[\hat{\boldsymbol{\omega}}_i^T \left(H H^T \right) \hat{\boldsymbol{\omega}}_i \right]^{\pm 1} + \sum_{j=s+1}^{z} \gamma_j \left[\hat{\dot{\boldsymbol{x}}}_j^T \left(H H^T \right)^{-1} \hat{\dot{\boldsymbol{x}}}_j \right]^{\pm 1} \tag{42}$$

with s being the number of directions with specified force requirements, $z - s$ the number of directions with specified velocity requirements, the exponent $+1$ is used in the directions where the force or velocity magnitude should be high, while the exponent -1 is used in the directions where there are requirements of precise velocity or force control, and γ_i , γ_j are factors to weight the relative importance of each magnitude and precision requirement in the respective directions. The configuration that maximizes this index allows to better apply forces and velocities in the directions of each of the desired grasp requirement.

5 Global quality measures

The criteria presented above measure the grasp quality under different considerations, based either on the location of contact points on the object or on the hand configuration. However, the optimal grasp for some particular tasks could be a combination of these considerations; as an extreme illustrative case, the selection of optimal contact points on the object surface according to any criteria from Section 3 ignoring the actual geometry of the hand can lead to contact locations unreachable for the real hand, and vice versa, an optimal hand configuration can generate a weak grasp in front of small perturbations. In order to deal with these situations, a global quality measure obtained through the combination of several criteria can be used, either combining in a serial or in a parallel way.

The serial approach is applied in grasp synthesis, using one of the quality criteria to generate candidate grasps and, from them, the best candidate is chosen using another quality measure. For instance, the optimization with respect to the hand configuration using the weighted sum in the task compatibility index given by equation (42) generates a preliminary grasp that, subsequently, is used as initial one for the search of an optimum grasp under the measure of the largest ball given by equation (25) [66].

The parallel approach combines different quality measures in a single global index. A simple method uses the algebraic sum of the quality resulting from each individual criterion, considering that all of them have to be either maximized or minimized. This approach has been used to choose optimum grasps of 2D objects [67]. A variation of this approach normalizes the outcome of each criterion dividing it by the difference between the measures of the best and the worst grasp. This approach has been used to evaluate grasps of 2D objects done with a three-finger hand [34], and different global measures are obtained by adding different basic criteria [35]. This can be interpreted as the generation of indexes specifically adapted for different practical applications. Another variation of these global indexes could be obtained by doing a weighted sum of different quality measures.

Kim et al. [68] use five normalized quality measures, covering the uncertainty in finger positions, the maximum force transmission ratio and the grasp isotropy, as well as a measure of the stability and another one related with the contact stiffness. Subsequently, a global quality index is defined as the minimum value out of the five normalized ones, weighted in agreement with the desired grasp properties.

Another possibility for combining in a parallel way different criteria is to rank grasps according to each of the measures, and later combine the rankings into a single one (taking into account the place for a grasp in each one of the rankings), to obtain the global optimal grasp; however, this approach has a high computational cost and has not given satisfactory outcomes [34].

6 Conclusions

This report has presented different grasp quality measures applicable to synthesis and evaluation of grasps. Table 2 summarizes the basic quality measures presented in this work. The quality measures have been classified in two large groups: measures associated with the location of contact points and measures associated with the hand configuration. Most of the quality measures presented in the literature are associated with the location of contact points, so we have divided this group into three subgroups. The first one contains measures based on algebraic properties of G; they have limited practical application because they do not consider any restriction on the forces applied in the contact points. The second subgroup considers the measures based on geometric relations of grasp, which are mostly used in the synthesis of independent contact regions. The third subgroup contains measures that consider limitation on finger forces, and includes one of the most used criteria in recent works on grasp synthesis (the criterion of the largest ball and its variations). The second group of quality measures includes criteria to get appropriate hand configurations for the grasp. A proper grasp should be optimal with respect to both groups of quality measures, and different global quality indexes have been proposed to simultaneously quantify the grasp with respect to both groups.

Although there are some studies comparing the optimal grasps obtained according to different criteria for different objects in bidimensional [59, 69, 70] and tridimensional grasps [53], the selection of the proper criterion in each real case is not always trivial. Also, even knowing the criterion to be applied, the complexity of real cases makes the computational cost of any grasp optimization to be really high most of the times. Finding the optimal grasp for real applications, in terms of the grasp quality and with an acceptable computational cost, is still an open research problem.

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References

[1] F. Lotti and G. Vassura, "Design aspects for advanced robot hands: Mechanical design," *Tutorial T1 - Towards Intelligent Robotic Manipulation, IEEE/RSJ IROS 2002* (2002), pp. 1–16.

[2] R.A. Grupen, T.C. Henderson and I.D. McCammon, "A survey on general purpose manipulation," *Int. J. Robotics Research* **8**(1), 38–62 (1989).

[3] A. Bicchi, "Hands for dexterous manipulation and robust grasping: A difficult road towards simplicity," *IEEE Trans. Robotics and Automation* **16**(6), 652–662 (2000).

[4] J. Tegin and J. Wikander, "Tactile sensing in intelligent robotic manipulation - a review," *Industrial Robot* **32**(1), 64–70 (2005).

[5] A.M. Okamura, N. Smaby and M.R. Cutkosky, "An overview of dexterous manipulation," *Proc. IEEE ICRA 2000* (2000), pp. 255–262.

[6] R.C. Brost and K.Y. Goldberg, "A complete algorithm for designing planar fixtures using modular components," *IEEE Trans. Robotics and Automation* **12**(1), 31–46 (1996).

[7] M.Y. Wang, "An optimum design for 3-D fixture synthesis in a point set domain," *IEEE Trans. Robotics and Automation* **16**(6), 839–846 (2000).

[8] B. Mishra and N. Silver, "Some discussion of static gripping and its stability," *IEEE Trans. on Systems, Man, and Cybernetics* **19**(4), 783–796 (1989).

[9] R.M. Murray, Z. Li and S. Sastry, *A Mathematical Introduction to Robotic Manipulation* (CRC Press, Boca Ratón, Florida, 1994).

Table 2. Grasp quality measures.

Group	Subgroup	Quality measure	Criterion	Subsect.
Measures associated with the position of the contact points	Measures based on algebraic properties of G	Minimum singular value of G - Force transmission ratio	Maximize	3.1.1
		Volume of the ellipsoid in the wrench space	Maximize	3.1.2
		Grasp isotropy index	Maximize	3.1.3
	Measures based on geometric relations	Stability grasp index	Minimize	3.2.1
		Area of the grasp polygon	Maximize	3.2.2
		Distance between the centroid C and the center of mass CM	Minimize	3.2.3
		Hypersphere radius in the force closure space	Maximize	3.2.4
		Minimum size of the independent contact region	Maximize	3.2.5
		Uncertainty grasp index	Minimize	3.2.5
	Measures considering limitations on the finger forces	Largest ball contained in $\mathcal{P}$	Maximize	3.3.1
		Largest ball with respect to all the reference systems	Maximize	3.3.2
		Volume of the convex hull $\mathcal{P}$	Maximize	3.3.2
		Q distance	Minimize	3.3.2
		Decoupled wrenches	Maximize	3.3.2
		Sum of the normal components in the finger forces	Minimize	3.3.3
		Distance of forces to the limits of the friction cone	Maximize	3.3.3
		Biggest ellipsoid $\mathcal{E}$ inscribed in $\mathcal{P}$	Maximize	3.3.4
Measures associated with the hand configuration		Smallest singular value of H	Maximize	4.1
		Volume of the manipulability ellipsoid	Maximize	4.2
		Condition number of H	Minimize	4.3
		Finger joint angle deviations	Minimize	4.4
		Task compatibility index	Maximize	4.5

[10] A. Bicchi, "On the closure properties of robotic grasping," *Int. J. Robotics Research* **14**(4), 319–344 (1995).

[11] B. Mishra, "Grasp metrics: Optimality and complexity," K. Goldberg, D. Halperin, J.C. Latombe and R. Wilson, editors, *Algorithmic Foundations of Robotics* (A.K. Peters, 1995), pp. 137–166.

[12] K.B. Shimoga, "Robot grasp synthesis algorithms: A survey," *Int. J. Robotics Research* **15**(3), 230–266 (1996).

[13] A. Bicchi and V. Kumar, "Robotic grasping and contact: A review," *Proc. IEEE ICRA 2000* (2000), pp. 348–352.

[14] M.R. Cutkosky, "On grasp choice, grasp models, and the design of hands for manufacturing tasks," *IEEE Trans. Robotics and Automation* **5**(3), 269–279 (1989).

[15] E. Rimon and J. Burdick, "On force and form closure for multiple finger grasps," *Proc. IEEE ICRA 1996* (1996), pp. 1795–1800.

[16] J. Kerr and B. Roth, "Analysis of multifingered robot hands," *Int. J. Robotics Research* **4**(4), 3–17 (1986).

[17] M. Buss, H. Hashimoto and J.B. Moore, "Dextrous hand grasping force optimization," *IEEE Trans. Robotics and Automation* **12**(3), 406–418 (1996).

[18] Y.H. Liu, "Qualitative test and force optimization of 3-D frictional form-closure grasps using linear programming," *IEEE Trans. Robotics and Automation* **15**(1), 163–173 (1999).

[19] G. Liu, J. Xu and Z. Li, "On geometric algorithms for real-time grasping force optimization," *IEEE Trans. Control Systems Technology* **12**(6), 843–859 (2004).

[20] W.S. Howard and V. Kumar, "On the stability of grasped objects," *IEEE Trans. Robotics and Automation* **12**(6), 904–917 (1996).

[21] Q. Lin, J. Burdick and E. Rimon, "A quality measure for compliant grasps," *Proc. IEEE ICRA 1997* (1997), pp. 86–92.

[22] H. Bruyninckx, S. Demey and V. Kumar, "Generalized stability of compliant grasps," *Proc. IEEE ICRA 1998* (1998), pp. 2396–2402.

[23] R.D. Howe, I. Kao and M.R. Cutkosky, "The sliding of robot fingers under combined torsion and shear loading," *Proc. IEEE ICRA 1988* (1988), pp. 103–105.

[24] M. Buss, H. Hashimoto and J.B. Moore, "Grasping force optimization for multi-fingered robot hands," *Proc. IEEE ICRA 1995* (1995), pp. 1034–1039.

[25] U. Helmke, K. Huper and J.B. Moore, "Quadraticaly convergent algorithms for optimal dextrous hand grasping," *IEEE Trans. Robotics and Automation* **18**(2), 138–146 (2002).

[26] N. Xydas and I. Kao, "Modeling of contact mechanics and friction limit surfaces for soft fingers in robotics with experimental results," *Int. J. Robotics Research* **18**(9), 941–950 (1999).

[27] S.L. Chiu, "Control of redundant manipulators for task compatibility," *Proc. IEEE ICRA 1987* (1987), pp. 1718–1724.

[28] T. Yoshikawa, "Analysis and control of robot manipulators with redundancy," *Proc. 1st Int. Symposium of Robotic Research* (1984), pp. 735–747.

[29] Z. Li and S. Sastry, "Task-oriented optimal grasping by multifingered robotic hands," *Proc. IEEE ICRA 1987* (1987), pp. 389–394.

[30] B. Kim, S. Oh, B. Yi and I.H. Suh, "Optimal grasping based on non-dimensionalized performance indices," *Proc. IEEE IROS 2001* (2001), pp. 949–956.

[31] Z. Li and S. Sastry, "Task-oriented optimal grasping by multifingered robotic hands," *IEEE J. Robotics and Automation* **4**(1), 32–44 (1988).

[32] Y.C. Park and G.P. Starr, "Grasp synthesis of polygonal objects using a three-fingered robotic hand," *Int. J. Robotics Research* **11**(3), 163–184 (1992).

[33] B. Mirtich and J. Canny, "Easily computable optimum grasps in 2D and 3D," *Proc. IEEE ICRA 1994* (1994), pp. 739–747.

[34] E. Chinellato, R.B. Fisher, A. Morales and A.P. del Pobil, "Ranking planar grasp configurations for a three-finger hand," *Proc. IEEE ICRA 2003* (2003), pp. 1133–1138.

[35] E. Chinellato, A. Morales, R.B. Fisher, and A.P. del Pobil, "Visual quality measures for characterizing planar robot grasps," *IEEE Trans. Systems, Man and Cybernetics - Part C: Applications and Reviews* **35**(1), 30–41 (2005).

[36] J. Ponce and B. Faverjon, "On computing three-finger force-closure grasps of polygonal objects," *IEEE Trans. Robotics and Automation* **11**(6), 868–881 (1995).

[37] J. Ponce, S. Sullivan, A. Sudsang, J.D. Boissonat and J.P. Merlet, "On computing four-finger equilibrium and force-closure grasps of polyhedral objects," *Int. J. Robotics Research* **16**(1), 11–35 (1997).

[38] D. Ding, Y.H. Liu and S. Wang, "Computation of 3-D form-closure grasps," *IEEE Trans. Robotics and Automation* **17**(4), 515–522 (2001).

[39] J. Cornellà and R. Suárez, "Fast and flexible determination of force-closure independent regions to gras polygonal objects," *Proc. IEEE ICRA 2005* (2005), pp. 778–783.

[40] Y.H. Liu, "Computing n-finger form-closure grasps on polygonal objects," *Int. J. Robotics Research* **19**(2), 149–158 (2000).

[41] Y. Li, Y. Yu and S. Tsujio, "An analytical grasp planning on given object with multi-fingered hand," *Proc. IEEE ICRA 2002* (2002), pp. 3749–3754.

[42] V.D. Nguyen, "Constructing force-closure grasps," *Int. J. Robotics Research* **7**(3), 3–16 (1988).

[43] D. Stam, J. Ponce and B. Faverjon, "A system for planning and executing two-finger force-closure grasps on curved 2-D objects," *Proc. IEEE/RSJ IROS 1992* (1992), pp. 210–217.

[44] J. Cornellà and R. Suárez, "Determining independent grasp regions on 2D discrete objects," *Proc. IEEE/RSJ IROS 2005* (2005), pp. 2936–2941.

[45] N.S. Pollard, "Synthesizing grasps form generalized prototypes," *Proc. IEEE ICRA 1996* (1996), pp. 2124–2130.

[46] Ch. Borst, M. Fischer and G. Hirzinger, "A fast and robust grasp planner for arbitrary 3D objects," *Proc. IEEE ICRA 1999* (1999), pp. 1890–1896.

[47] D.G. Kirkpatrick, B. Mishra and C.K. Yap, "Quantitative Steinitz's theorem with applications to multifingered grasping," *Discrete and Computational Geometry* **7**(3), 295–318 (1992).

[48] C. Ferrari and J. Canny, "Planning optimal grasps," *Proc. IEEE ICRA 1992* (1992), pp. 2290–2295.

[49] J. Cornellà and R. Suárez, "On 2D 4-finger frictionless optimal grasps," *Proc. IEEE/RSJ IROS 2003* (2003), pp. 3680–3685.

[50] Ch. Borst, M. Fischer and G. Hirzinger, “Grasping the dice by dicing the grasp,” *Proc. IEEE/RSJ IROS 2003* (2003), pp. 3692–3697.

[51] A.T. Miller and P.K. Allen, “GraspIt! a versatile simulator for robotic grasping,” *IEEE Robotics and Automation Magazine* **11**(4), 110–122 (2004).

[52] M. Teichmann, “A grasp metric invariant under rigid motions,” *Proc. IEEE ICRA 1996* (1996), pp. 2143–2148.

[53] A.T. Miller and P.K. Allen, “Examples of 3D grasp quality computations,” *Proc. IEEE ICRA 1999* (1999), pp. 1240–1246.

[54] X. Zhu, H. Ding and H. Li, “A quantitative measure for multifingered grasps,” *Proc. IEEE/ASME Int. Conf. Advanced Intelligent Mechatronics 2001* (2001), pp. 213–219.

[55] X. Zhu and J. Wang, “Synthesis of force-closure grasps on 3-D objects based on the Q distance,” *IEEE Trans. Robotics and Automation* **19**(4), 669–679 (2003).

[56] G. Liu, J. Xu, X. Wang and Z. Li, “On quality functions for grasp synthesis, fixture planning and coordinated manipulation,” *IEEE Trans. Automation Science and Engineering* **1**(2), 146–162 (2004).

[57] N.S. Pollard, “Closure and quality equivalence for efficient synthesis of grasps from examples,” *Int. J. Robotics Research* **23**(6), 595–614 (2004).

[58] L. Han, J.C. Trinkle and Z.X. Li, “Grasp analysis as linear matrix inequality problems,” *IEEE Trans. Robotics and Automation* **16**(6), 663–674 (2000).

[59] Ch. Borst, M. Fischer and G. Hirzinger, “Grasp planning: How to choose a suitable task wrench space,” *Proc. IEEE ICRA 2004* (2004), pp. 319–325.

[60] R. Haschke, J.J. Steil, I. Steuwer and H. Ritter, “Task-oriented quality measures for dextrous grasping,” *Proc. 6th IEEE Int. Conf. Computational Intelligence in Robotics and Automation* (2005), pp. 689–694.

[61] C.A. Klein and B.E. Blaho, “Dexterity measures for the design and control of kinematically redundant manipulator,” *Int. J. Robotics Research* **6**(2), 72–83 (1987).

[62] T. Yoshikawa, “Manipulability of robotic mechanisms,” *Int. J. Robotics Research* **4**(2), 3–9 (1985).

[63] J.K. Salisbury and J.J. Craig, “Articulated hands: Force control and kinematic issues,” *Int. J. Robotics Research* **1**(1), 4–17 (1982).

[64] A. Liegeois, “Automatic supervisory control for the configuration and behavior of multibody mechanisms,” *IEEE Trans. System, Man and Cybernetics* **7**(12), 842–868 (1977).

[65] S.L. Chiu, “Task compatibility of manipulator postures,” *Int. J. Robotics Research* **7**(5), 13–21 (1988).

[66] R.D. Hester, M. Cetin, C. Kapoor and D. Tesar, “A criteria-based approach to grasp synthesis,” *Proc. IEEE ICRA 1999* (1999), pp. 1255–1260.

[67] E. Boivin, I. Sharf and M. Doyon, “Optimum grasp of planar and revolute objects with gripper geometry constraints,” *Proc. IEEE ICRA 2004* (2004), pp. 326–332.

[68] B. Kim, B. Yi, S. Oh and I.H. Sung, “Non-dimensionalized performance indices based optimal grasping for multi-fingered hands,” *Mechatronics* **14**(3), 255–280 (2004).

[69] G.M. Bone and Y. Du, “Multi-metric comparison of optimal 2D grasp planning algorithms,” *Proc. IEEE ICRA 2001* (2001), pp. 3061–3066.

[70] A. Morales, P.J. Sanz, A.P. del Pobil and A.H. Fagg, “An experiment in constraining vision-based finger contact selection with gripper geometry,” *Proc. IEEE/RSJ IROS 2002* (2002), pp. 1693–1698.